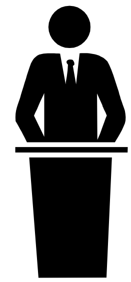
## Modeling of Economic Series Coordinated with Interest Rate Scenarios



***Research Sponsored by the Casualty Actuarial Society and the Society of Actuaries***

***<u>Investigators:</u>***
Kevin Ahlgrim, ASA, PhD, Illinois State University
Steve D'Arcy, FCAS, PhD, University of Illinois
Rick Gorvett, FCAS, ARM, FRM, PhD, University of Illinois

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# Acknowledgements

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# Outline of Presentation

- Motivation for Financial Scenario Generator Project
- Short description of included economic variables
- An overview of the model
- Applications of the model
- Comparison of this model with another actuarial return generating model
- Conclusions

# Overview of Project

- CAS/SOA Request for Proposals on "**Modeling of Economic Series Coordinated with Interest Rate Scenarios"**
  - A key aspect of ***dynamic financial analysis***
  - Also important for regulatory, rating agency, and internal management tests – e.g., ***cash flow testing***
- Goal: to provide actuaries with a model for ***projecting economic and financial indices***, ***with realistic interdependencies*** among the variables.
  - Provides a ***foundation*** for future efforts

## Scope of Project

- ***Literature review***
  - From finance, economics, and actuarial science
- ***Financial scenario model***
  - Generate scenarios over a 50-year time horizon
- ***Document and facilitate use of model***
  - Report includes sections on data & approach, results of simulations, user's guide
  - Posted on CAS & SOA websites
  - Writing of papers for journal publication

## Economic Series Modeled

- Inflation
- Real interest rates
- Nominal interest rates
- Equity returns
  - Large stocks
  - Small stocks
- Equity dividend yields
- Real estate returns
- Unemployment

# Current Report Structure

***Text Sections***

1) Intro & Overview
2) Excerpts from RFP
3) Selected Proposal
4) Literature Review
5) Data & Approach
6) Issue Discussion
7) Results of Simulations
8) Conclusion

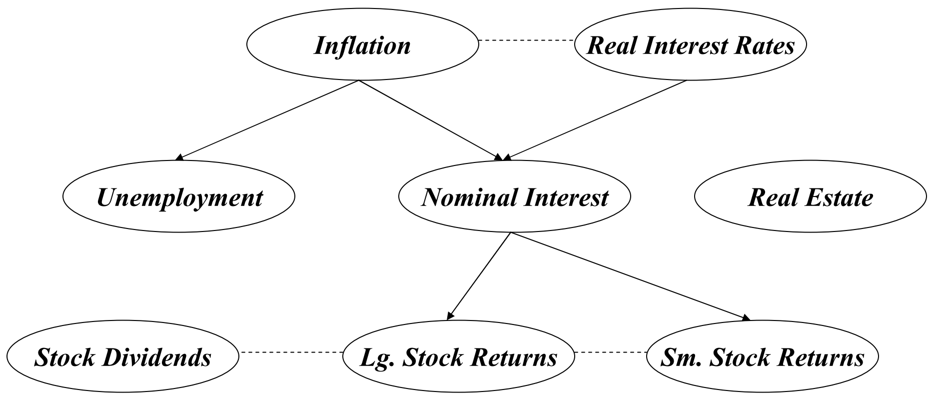
***Appendices***

A) User's Guide to the Model
B) Presentations of this Research
C) Simulated Financial Scenario Data
D) Financial Scenario Model

# Prior Work

- Wilkie, 1986 and 1995
  - Widely used internationally
- Hibbert, Mowbray, and Turnbull, 2001
  - Modern financial tool
- CAS/SOA project (a.k.a. the Financial Scenario Generator) applies Wilkie/HMT to U.S.

# Relationship between Modeled Economic Series




# Inflation ($q$)

- Modeled as an Ornstein-Uhlenbeck process
  - One-factor, mean-reverting

$$dq_t = \kappa_q (\mu_q - q_t)\, dt + \sigma\, dB_q$$

- Speed of reversion: $\kappa_q = 0.40$
- Mean reversion level: $\mu_q = 4.8\%$
- Volatility: $\sigma_q = 0.04$

# Explanation of the Ornstein-Uhlenbeck process

- Deterministic component

  *If inflation is below 4.8%, it reverts back toward 4.8% over the next year*

  *Speed of reversion dependent on* $\kappa$
- Random component

  *A shock is applied to the inflation rate that is a random distribution with a std. dev. of 4%*
- The new inflation rate is last period's inflation rate changed by the combined effects of the deterministic and the random components.

# Real Interest Rates ($r$)

- Problems with one-factor interest rate models
- Two-factor Vasicek term structure model
- Short-term rate (***r***) and long-term mean (***l***) are both stochastic variables

$$dr_t = \kappa_r (l_t - r_t)\, dt + \sigma_r\, dB_r$$

$$dl_t = \kappa_l (\mu_l - r_t)\, dt + \sigma_l\, dB_l$$

# Nominal Interest Rates

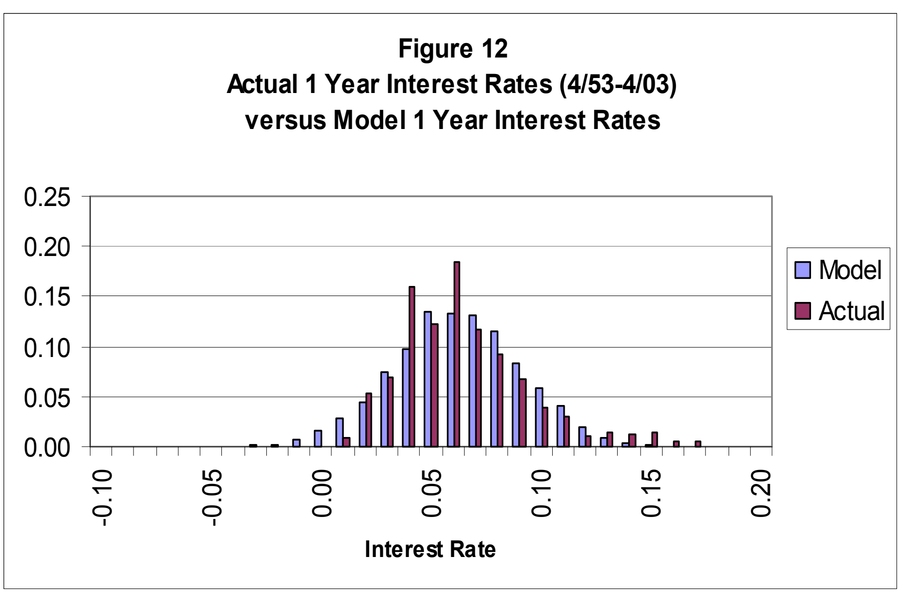
- Combines inflation and real interest rates

$$i = \{(1+q) \times (1+r)\} - 1$$

where $i$ = nominal interest rate

$q$ = inflation

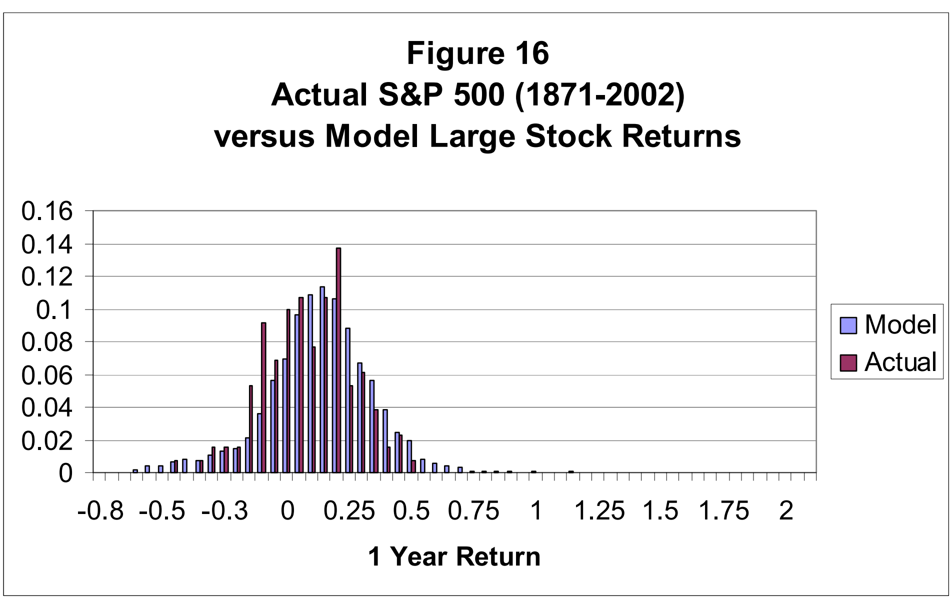
$r$ = real interest rate

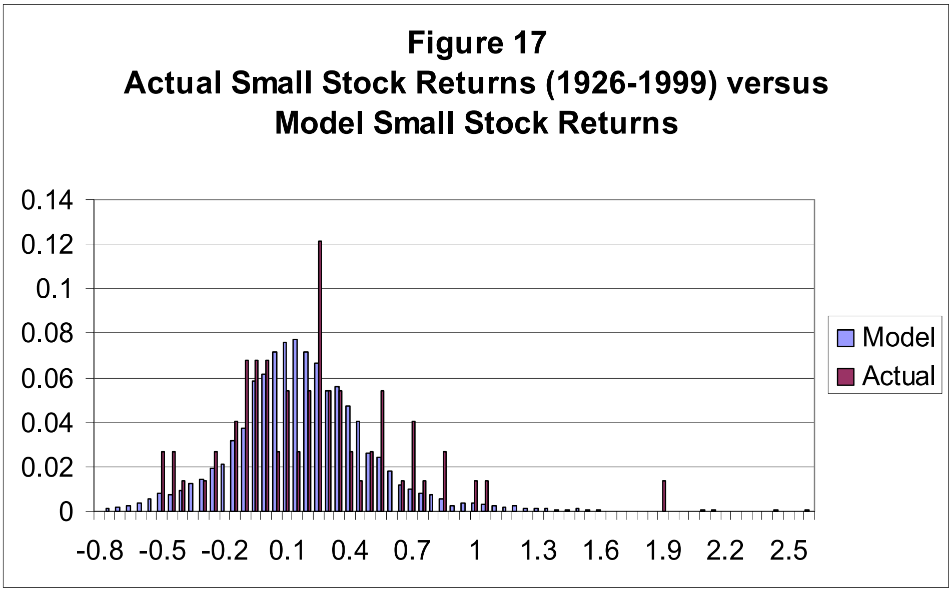


# Equity Returns

- Empirical "fat tails" issue regarding equity returns distribution
- Thus, modeled using a "regime switching model"
  1. High return, low volatility regime
  2. Low return, high volatility regime
- Model equity returns as an excess return (***$x_t$***) over the nominal interest rate

$$s_t = q_t + r_t + x_t$$






# Other Series

- ***Equity dividend yields*** (***y***) ***and real estate***
  - Mean-reverting processes
- ***Unemployment*** (***u***)
  - Phillip's curve: inverse relationship between ***u*** and ***q***

$$du_t = \kappa_u (\mu_u - u_t)\, dt + \alpha_u\, dq_t + \sigma_u\, \varepsilon_{ut}$$

## Selecting Parameters

- Historical or calibration with current market prices
- Model is meant to represent range of outcomes possible for the insurer
- Default parameters are chosen from history (as long as possible)
- Of course, different parameters may affect analysis

## Model Description

- Excel spreadsheet
- Simulation package - @RISK add-in
- 50 years of projections
- Users can select different parameters and track any variable

## Applications of the Financial Scenario Generator

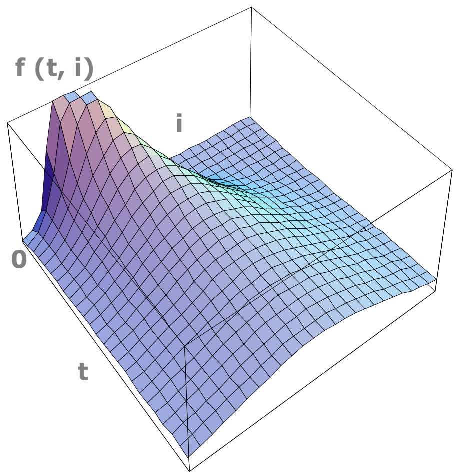
- Financial engine behind many types of analysis
- Insurers can project operations under a variety of economic conditions (Dynamic financial analysis)
- Useful for demonstrating solvency to regulators
- May propose financial risk management solutions

## CAS/SOA vs. AAA

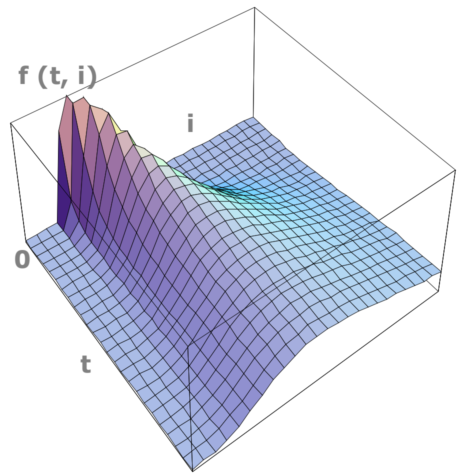
- AAA models provides guidance for Risk-Based Capital (RBC) requirements for variable products with guarantees
- Focus is on
  - Interest rate risk
  - Equity risk
- 10,000 Pre-packaged scenarios available
- Available at:

http://www.actuary.org/life/phase2.htm

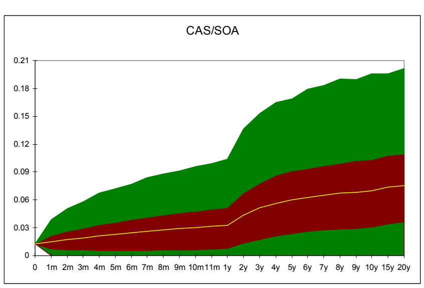
CAS/SOA

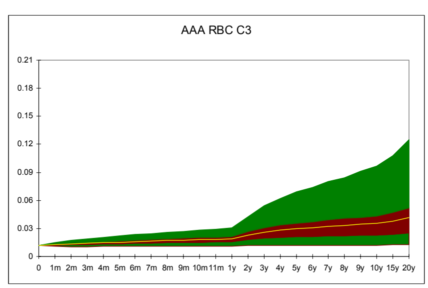



AAA RBC C3

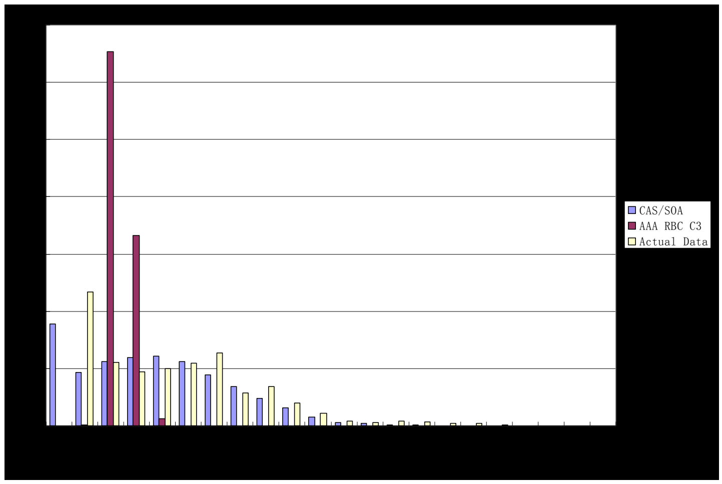



Funnel of Doubt Graphs
3 Month Nominal Interest Rates (U. S. Treasury Bills)

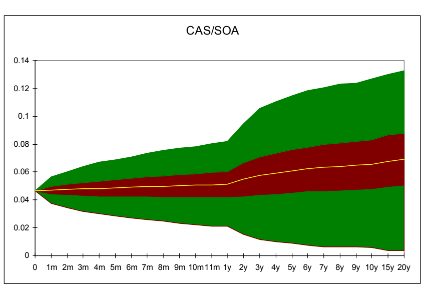


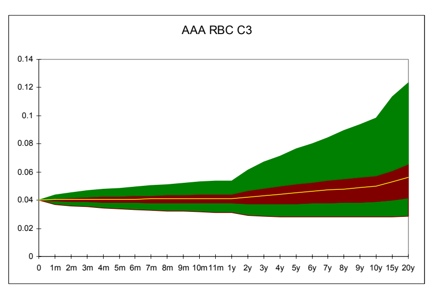



## Histogram of 3 Month Nominal Interest Rates
## Model Values and Actual Data (01/34-05-04)

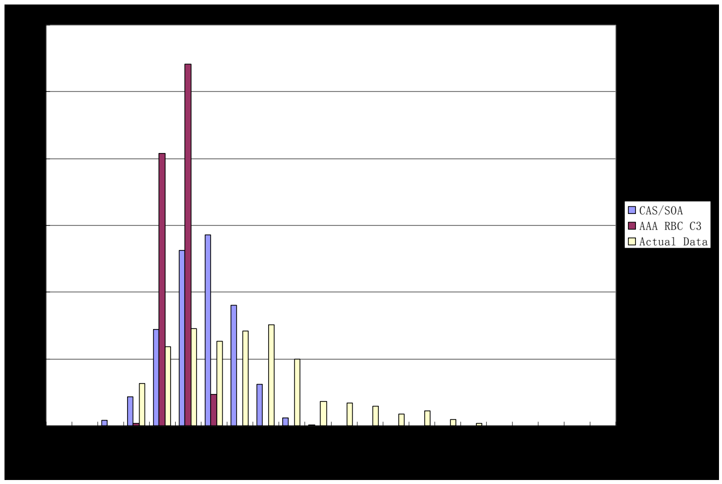



## Funnel of Doubt Graphs
## 10 Year Nominal Interest Rates (U. S. Treasury Bonds)

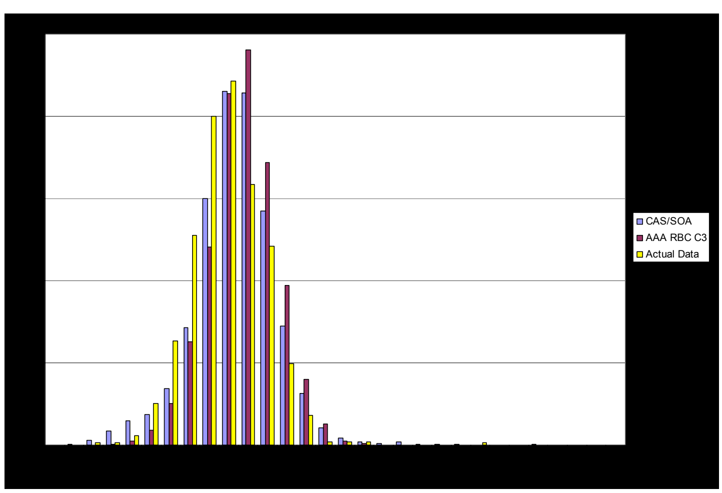


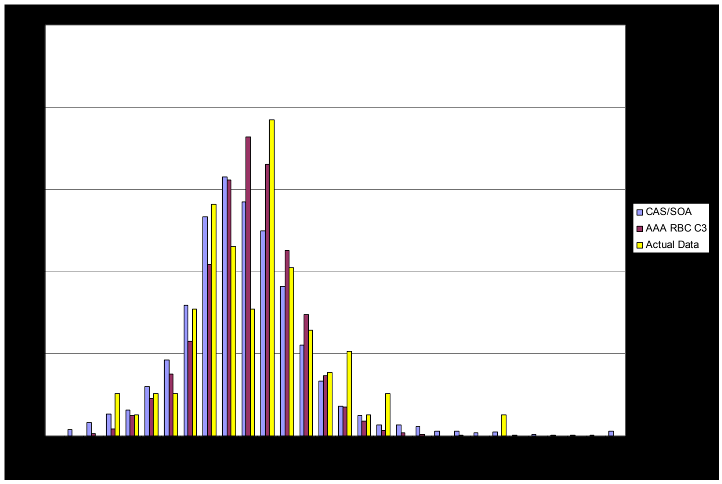
## Histogram of 10 Year Nominal Interest Rates
## Model Values and Actual Data (04/53-05/04)




## Histogram of Large Stock Return
## Model Values and Actual Data (1872-2004)



Histogram of Small Stock Return
Model Values and Actual Data (1926-2003)




# How to Obtain Model

Posted on the following sites:

- http://casact.org/research/econ/
- http://www.soa.org/ccm/content/areas-of-practice/finance/mod-econ-series-coor-int-rate-scen/

Or contact us at: kahlgrim@ilstu.edu
s-darcy@uiuc.edu
gorvett@uiuc.edu